

Semiotic Representation Fluency in Mathematics: An Overview

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ABSTRACT:

The semiotic process facilitates the ability of students to adeptly manipulate and comprehend mathematical signs and symbols. The understanding of these signs fosters a connection between mathematical objects and their respective interpretants. This underscores the critical significance of fluency in semiotic representation within the discipline of mathematics, which subsequently enhances students' cognitive capabilities in dealing with semiotic representations within a singular register as well as transitioning from one register of semiotic representation to another. These transformations of semiotic representation can be categorized into distinct forms, including semantic, symbolic, and graphic. This thematic exploration delineates these two categories of transformations through illustrative examples. Additionally, it elucidates several key components of semiotic representation fluency essential for achieving proficiency in semiotic representation, such as the comprehension of Representamen, transformations pertaining to semiotic representation, and the cognitive skills requisite for fluency in semiotic representation. The aforementioned essential elements are elucidated through the application of Pierce's Triadic model alongside the categorisation of

the four registers- namely, discursive, non- discursive, mono-functional and multi-functional registers- that can be engaged in mathematical activities.

KEYWORDS: Semiosis; Semiotic Representation Fluency; Representamen; Transformation Fluency; Mathematical Object; Cognitive Processes.

INTRODUCTION:

The challenges associated with comprehending mathematical concepts present significant difficulties for numerous students, thereby constituting a formidable challenge within the classroom environment. In order to discern the barriers impeding student comprehension, it is imperative to acknowledge the critical significance of proficiency in semiotic representation. The development of fluency in semiotic representation is a fundamental prerequisite for the development of mathematical thinking. In the realm of mathematical processing, no operations can be executed devoid of a semiotic representation system, as such processing invariably necessitates the substitution of one semiotic representation for another (Duval, 2006). All varieties of mathematical endeavours encompass semiotic representation. In order to scrutinize the fundamental components that contribute to the comprehension of mathematical activities as well as the obstacles that impede students' advancement in mathematical cognition and their interpretative abilities in resolving mathematical issues in a coherent manner, semiotic representation assumes a pivotal significance.

These semiotic representations are amenable to transformation across various registers. The mathematical process consists of two distinct transformations, namely treatment and conversion. Treatment occurs within a singular register (Duval, 2006). For instance, the treatment involves the semiotic representation of addition and subtraction of integers within the framework of mathematical operation properties. In the conversion between semiotic representations, the register transitions from the initial representation to the terminal representation. For example, the transformation of semiotic representation may occur from semantic to graphic or symbolic representation. This thematic investigation posited the notion

of semiotic representation fluency as a means to enhance mathematical understanding, drawing upon Pierce's triadic framework of representamen, Object and the Interpretant, alongside the various semiotic representations: Treatment and Conversion.

SEMIOTICS: CONCEPT AND MEANING

Semiotics represents an academic inquiry into the nature of signs and the behaviours associated with their utilization. This discipline was articulated by one of its seminal figures, the Swiss linguist Ferdinand de Saussure, who characterized it as the exploration of "the life of signs within society" ("Semiótica tricerebral", 2022). While the term was employed in such a context during the 17th century by the English philosopher John Locke, the conceptualization of semiotics as an interdisciplinary domain of scholarly investigation only materialized in the late 19th and early 20th centuries through the autonomous contributions of Saussure and the American philosopher Charles Sanders Peirce. The Editors of Encyclopaedia Britannica. (2025, July 4).

Semiotics means the scholarly examination of signs to interpret and manipulate mathematical concepts. It is essential in Mathematics to comprehend the mathematical constructs, thereby enhancing the mathematical problem solving ability of the students. This field supports students in articulating mathematical ideas and augments their cognitive abilities. The manifestations of these signs may encompass symbolic, visual, pictorial, or graphic representations of mathematical notions, as well as their semantic interpretations or cognitive constructs that signify mathematical entities. The semiotic process further fosters comprehension through the manipulation and organization of mathematical symbols (Purwasih et al., 2023). For instance, when addressing the equations $2x + y = 12$ and $x - y = 3$, we employ mathematical principles to ascertain the pairs of numbers that concurrently satisfy these two equations through a series of logical steps involving the manipulation of the symbols x and y . Through this semiotic methodology, we endeavour to understand that there exists a singular solution to the aforementioned linear equations, thereby ascribing values or meanings to the mathematical symbols x and y . Moreover, this semiotic process cultivates a

profound understanding of the diverse mathematical principles and manipulations pertaining to these symbols, such as x and y , and correlates them to the human perception of mathematical realities.

SEMIOTIC REPRESENTATION FLUENCY IN MATHEMATICS:

In the field of mathematical semiosis, the “representamen” links the “interpretant” to the mathematical “object” that it refers to (Palayukan, 2022).

The phenomenon of semiotic transpires when learners engage with symbols or signs pertinent to mathematics to facilitate the understanding, representation, application, and generation of the mathematical knowledge and concepts they have acquired in novel and unfamiliar contexts. For instance, the expression $x > y$ employs the representamen ($>$) to establish a relationship between the mathematical entities x and y . Moreover, the interpretant serves to

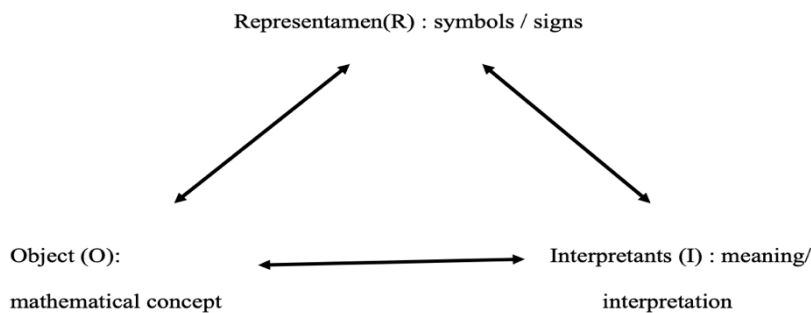


Fig: 1 (Charles Pierce's Triadic Concept)

link the representamen with the object, thereby facilitating the comprehension of the notion that the value of x exceeds that of y . Semiosis constitutes a cognitive process. Consequently, semiotics can enhance the efficacy and efficiency of mathematics instruction and learning (Palayukan et al., 2020).

This triadic concept facilitates the semiotic process in conducting symbolic, semantic, visual, or diagrammatic representations of mathematical concepts. Such semiotic representations are instrumental in aiding students to effectively comprehend, interpret, and manipulate mathematical objects. Semiotic representations function as instruments for accessing mathematical concepts and forging connections between diverse forms of representation

(Duval, 2017). They promote the advancement of mathematical reasoning by enabling students to adeptly manipulate and interpret symbols (Castillo & Alarcón, 2014).

The students are acquiring substantial opportunities to engage with mathematical concepts through the utilization of semiotic representations (Iori, 2017). Furthermore, the comprehension of the subject matter is facilitated through the transformation and interpretation of mathematical concepts via semiotic representations (Fandino, 2010).

According to Duval (2006), the semiotic representation within the domain of mathematics is categorized into four distinct semiotic registers: discursive, non-discursive, monofunctional, and multifunctional registers. The discursive registers encompass both natural languages and symbolic systems that convey oral explanations alongside visual representations. This category entails the denotation of objects, the articulation of relations or properties, and inferential processes such as deduction, computation, or proof. The non-discursive registers signify iconic representations, including drawings, sketches, and patterns, as well as non-iconic representations such as geometrical figures that can be constructed using tools, diagrams, and graphs. In monofunctional registers, the majority of processes are algorithmic, adhering to prescribed rules of representation formation; conversely, processes within multifunctional registers resist reduction to algorithmic forms. This classification serves to assist educators in analysing the comprehension of mathematical activities and in identifying the comprehension difficulties that numerous students encounter.

“ Some semiotic systems can be used for only one cognitive function: mathematical processing. On the other hand, other semiotic systems can fulfil a large range of cognitive

functions: communication, information processing, awareness, imagination, etc”(Duval, 1995b). The heterogeneous characteristics of semiotic representation within the discipline of mathematics are paramount for individuals to engage in diverse cognitive activities, thereby attaining an enhanced level of conceptual comprehension of the subject matter.

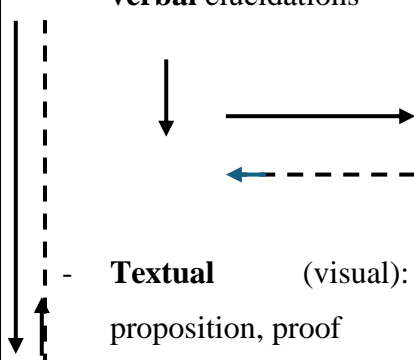

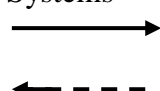
	DISCURSIVE REGISTERS	NON-DISCURSIVE REGISTERS
<p>MULTI-FUNCTIONAL REGISTERS: Processes cannot be made into algorithms</p>	<p>IN NATURAL LANGUAGE</p> <p>- verbal elucidations</p> <p>- Textual (visual): proposition, proof</p> 	<p>Illustration, diagram, design</p> <p>Abstract representations:</p> <p>Geometric constructs that can be fabricated using instruments.</p>
	<p>Representations supporting transition</p>	
<p>MONO-FUNCTIONAL REGISTERS: are algorithmic in nature</p>	<p>In representational Systems</p> <p>Written, computation, proof</p> 	<p>Diagrams, graphs, shapes</p>

Fig:2 Duval's classification of the registers of Semiotic Representations.

Fluency in semiotic representation within the realm of mathematics pertains to the capacity to represent mathematical objects, signs, and symbols, as well as the proficiency to interpret and manipulate mathematical concepts in a manner conducive to logical comprehension and problem-solving. This fluency in semiotic representation bears considerable importance at the primary and secondary educational levels, as it fosters the development of students' cognitive faculties, including comprehension, logical reasoning, interpretative skills, analytical abilities, and visualization. Furthermore, fluency in semiotic representation is critical for the cultivation of constructive thinking.

COMPONENTS OF SEMIOTIC REPRESENTATION FLUENCY:

No type of mathematical manipulation can be executed without the application of a semiotic representational system, as mathematical manipulation invariably entails the exchange of one semiotic representation for another. The role of signs is to symbolize mathematical concepts, which is integral for facilitating their metamorphosis. In distinction from other fields of scientific exploration, the transformation of signs and semiotic representations holds a pivotal position in mathematical pursuits (Duval, 2006). Students exhibiting diminished fluency in semiotic representation may adversely affect their academic performance within the classroom environment, as well as their overall proficiency in the subject matter. Those with poor representation transformation skills are typically less proficient in applying mathematical concepts and relationships (Gagatsis & Shiakalli, 2004; Uzun & Arslan, 2009).

- **Comprehension of Representamen (sign/ symbol):**

A sign is defined as “something that stands for something else” (Colapietro 1993, p. 179). It is imperative for students to comprehend mathematical symbols along with their corresponding meanings, thereby equipping them to adeptly manipulate these symbols in contexts of problem-solving (Firnanda & Wahyuni, 2024). Such comprehension is crucial for

the development of skills related to representational transformation, which have a profound impact on students' efficacy in resolving mathematical challenges (Dahiana et al., 2023).

- **Transformation (Treatment and Conversion) Fluency:**

As articulated by Duval (1995), : “There’s no knowledge that can be mobilised by an individual without a representation activity”(P.15). There exist two distinct types of transformations concerning semiotic representations: treatment and conversion. Semiotic

treatments are transformations of representation within the same register(Duval,2006). More specifically, in treatment, the semiotic analysis commences with a symbolic representation and, subsequent to a transformative process, concludes with an alternative symbolic representation of an identical nature (Duval, 2006, 2017b). Should the initial transformation be situated within a semantic, symbolic, or graphical register, the resultant representation subsequent to the semiotic treatment by the interpretant will likewise manifest as another semantic, symbolic, or graphical register (Duval, 2006, 2017a).For instance, in the factorisation of a polynomial, $P(x) = 2x^2 - 5x + 3$


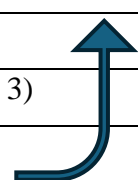
Initial representation : $P(x) = 2x^2 - 5x + 3$		symbolic register
Treatment 1 : splitting the middle term	$P(x) = 2x^2 - 3x - 2x + 3$	
Treatment 2: grouping the terms	$P(x) = x(2x - 3) - 1(2x - 3)$	
Treatment 3: common binomial	$P(x) = (2x - 3)(x - 1)$	
Terminal representation : $P(x) = (2x - 3)(x - 1)$		

Table:1 Semiotic treatment on Polynomial factorisation.

The conversion of semiotic representation is explicated as the transformation of one symbolic representation into another that possesses a distinctly different nature (Duval, 2017a, 2017b). This phenomenon of semiotic conversion encompasses the modification of a

register while maintaining the integrity of the mathematical object (Duval, 2006). More precisely, the semiotic conversion begins with a semantic representation or graphical or diagrammatic or symbolic representation and, subsequent to a transformative process, terminates with another representation that is recognizably different in characteristics. The ability to convert between different representations aids a person in developing skills necessary for handling intricate mathematical problems, allowing them to transition from one form to another. Additionally, this skill enhances their capacity to interpret, analyse and generate mathematical representations.

For example:

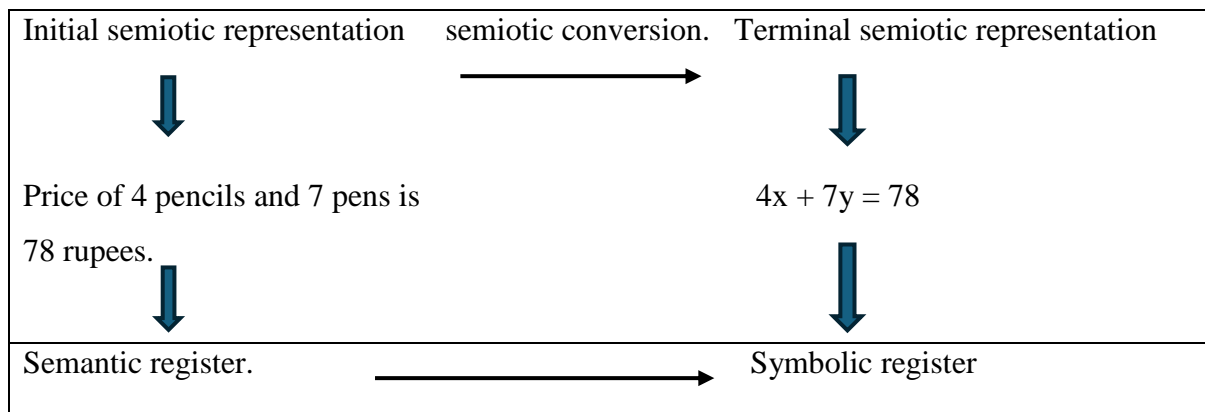
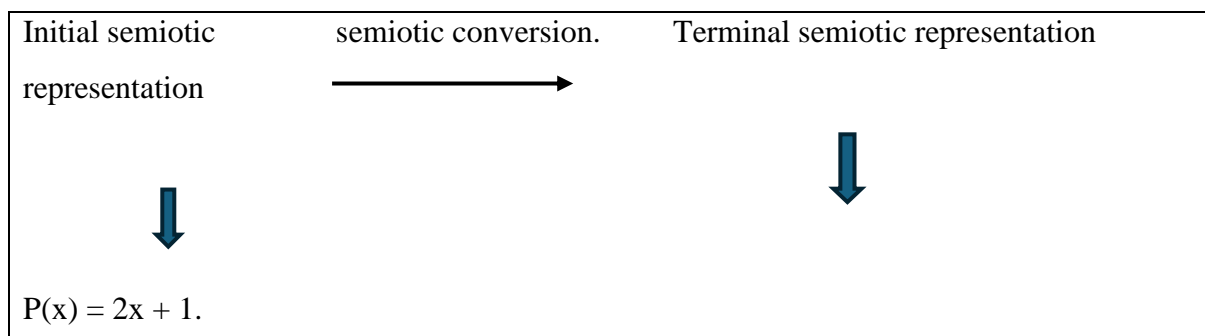


Table:2 Conversion of semantic register to symbolic register.



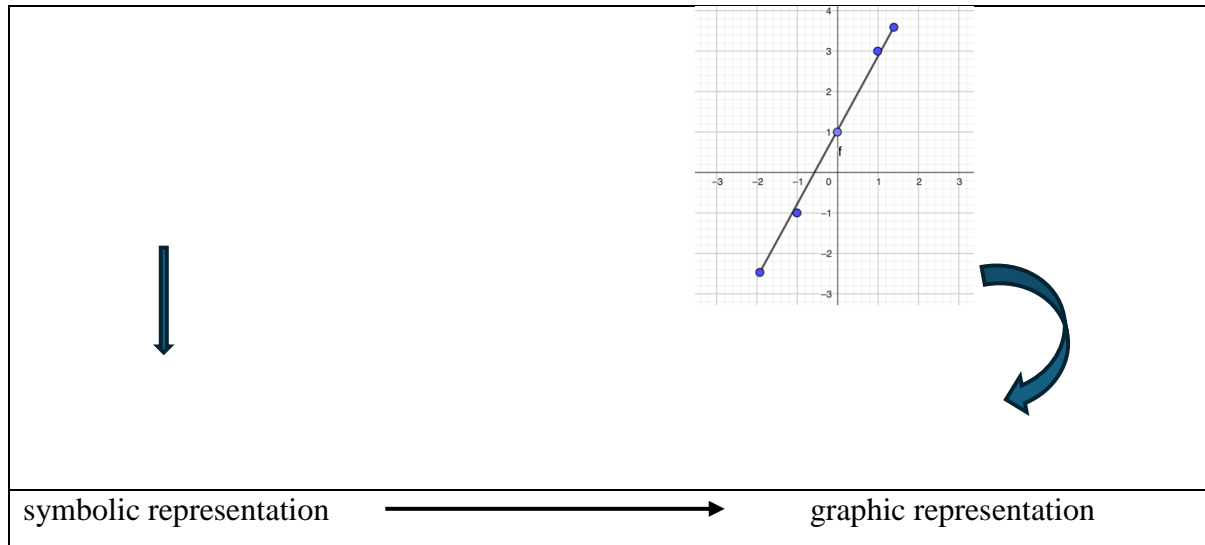


Table:3 Conversion of symbolic representation to graphic representation

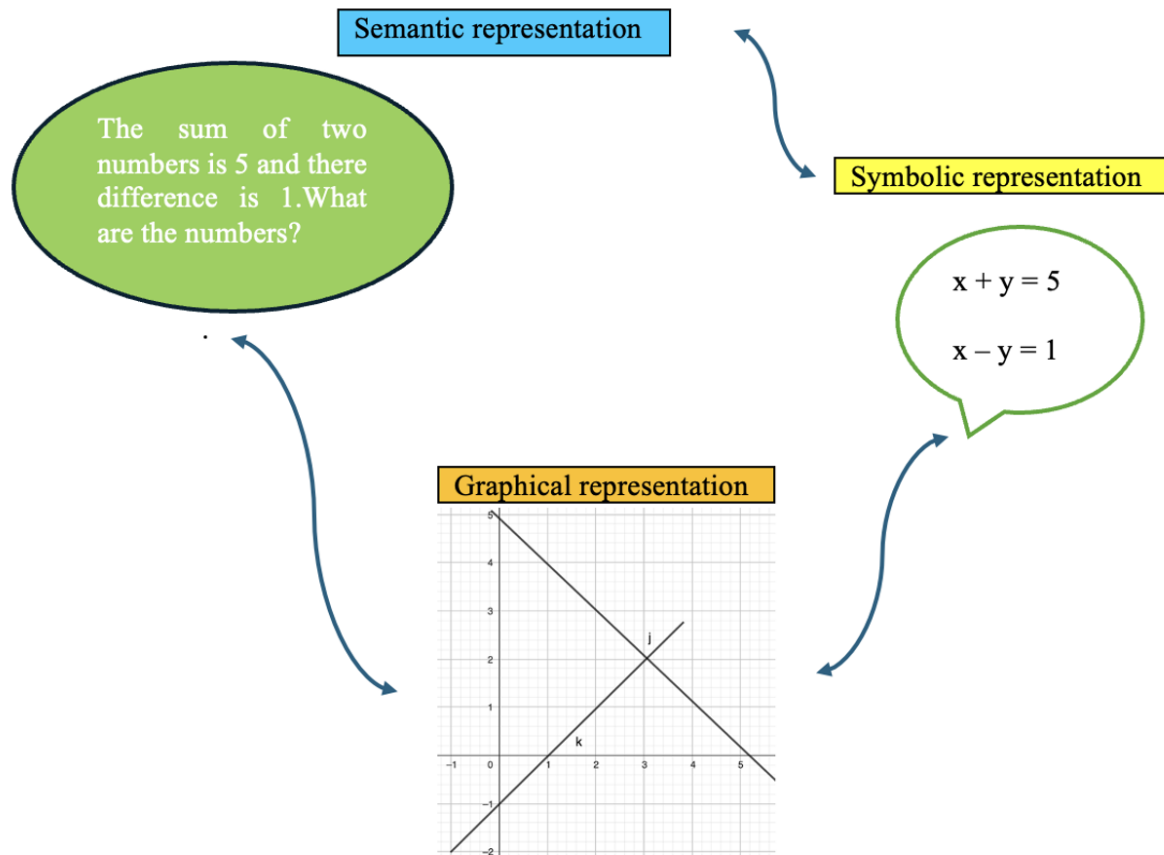


Fig:3 shows conversion between semiotic representations of pair of equations.

Figure 3 shows the possible three different semiotic conversions: semantic to symbolic, symbolic to graphic and graphic to semantic. The semiotic conversion of a pair of equations transformed semantically, graphically and symbolically. Duval (2017b) asserts that mathematical concepts remain invariant following their definition; nonetheless, engagement with these concepts necessitates the utilization of diverse semiotic representations. The capacity to transition between various forms of representation (e.g., algebraic to geometric) is paramount (Dahiana et al., 2023). Exceptional students possess the capacity to excel in various mathematical representations, whereas students of average ability may struggle to achieve similar proficiency.

- **Mathematical Object:**

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The term mathematical object pertains to the realm of mathematical concepts. The student should be able to comprehend, interpret and analyse these mathematical objects. According to Peirce's semiotic theory, students need to interpret mathematical objects and their relationships accurately. This includes understanding the problem context and deriving appropriate mathematical models (Susanti et al., 2023). A solid conceptual understanding facilitates students in articulating their mathematical ideas, employing diverse strategies for problem-solving, and making sense of mathematical scenarios.

- **Cognitive process:**

The semiotic process necessitates the application of cognitive skills. Transforming representations within a singular register, managing semiotic representations, and transitioning from one initial representation to another terminal representation necessitates a cognitive engagement with the mathematical objects and the requisite interpretive skills. The proficiency to convert from one semiotic representation to another is crucial for the comprehension of mathematical objects, which is vital for advancement in mathematical learning, the analysis and interpretation of mathematical concepts, and the cultivation of creativity in addressing mathematical problems. The cognitive constructs involved in any mathematical endeavour necessitate a focus on the cognitive strata in semiotic representation. Duval (1995, p. 17) elucidates this notion by remarking: "The notion of semiotic representation presupposes the consideration of different semiotic systems and a cognitive operation of conversion of representations from one system into another." This conversion entails mobilization from one register to another: transitioning from a semantic statement into symbolic or algebraic representations, or from algebraic to geometric forms.

CONCLUSION:

Semiosis or the utilization of signs occupies a pivotal position within the realm of mathematics. Signs and semiotic representations constitute an integral component of all mathematical endeavours, as well as the cognitive processes that encompass the

understanding of mathematical concepts, as underscored by Duval's cognitive model. According to Duval, mathematical activity transpires across four distinct registers—either within a singular register or among multiple registers. The transformation of semiotic representation occurring within a singular register is termed treatment (e.g., algebra to algebra), whereas the transition between registers—mobilizing from one register to another (e.g., algebra to geometry)—is referred to as conversion. To attain proficiency in mathematics, students must demonstrate competence in executing these two transitions: treatment and conversion. Such competencies necessitate the ability to represent mathematical objects and cognitive processes in various modalities, including visual, verbal, graphic, symbolic, diagrammatic, or figurative forms. Students often encounter less difficulty with treatment, yet they frequently struggle with conversion. The degree of fluency in semiotic representation significantly influences the comprehension capacity associated with any mathematical activity. Educators can facilitate students' recognition of the cognitive processes implicated in the conversion between registers—namely, semantic, symbolic, graphic, and figurative—thereby enhancing their productivity.

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